

1. Determine the domain and range of  $f^{-1}$  for the given function  $f$  without actually finding the inverse function.

$$f(x) = \frac{3}{2-3x}$$

Domain  $f$ :  $2-3x \neq 0$   
 $-3x \neq -2$   
 $x \neq \frac{2}{3}$

Range  $f^{-1}$ :  $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

Domain  $f^{-1}$ :  $(-\infty, 0) \cup (0, \infty)$

Range  $f$ : Fraction w/ constant numerator  $\rightarrow f(x) \neq 0$

2. Find the inverses of the following functions.

a)  $g(x) = 5 - 7x$

$$g(x) - 5 = -7x$$

$$\frac{g(x) - 5}{-7} = x, \text{ so:}$$

$g(x) \leftrightarrow y$   
 $g^{-1}(y) \leftrightarrow x$

$$\frac{y-5}{7} = g^{-1}(y)$$

b)  $h(x) = \frac{9x+5}{5x-6}$

$$h(x)(5x-6) = 9x+5$$

$$5x h(x) - 6h(x) = 9x+5$$

$$5x h(x) - 9x = 5 + 6h(x)$$

$$x(5h(x) - 9) = 5 + 6h(x)$$

$$x = \frac{5+6h(x)}{5h(x)-9}, \text{ so:}$$

$$h^{-1}(y) = \frac{5+6y}{5y-9}$$

c)  $k(x) = 4x^3 - 6$

$$k(x) + 6 = 4x^3$$

$$\frac{k(x)+6}{4} = x^3$$

$$\sqrt[3]{\frac{k(x)+6}{4}} = x$$

so:  $\sqrt[3]{\frac{y+6}{4}} = k^{-1}(y)$

3. Determine the inverse function of  $(g \circ f)(x)$ .

$$(g \circ f)^{-1} = (f^{-1} \circ g^{-1}) \quad \star$$

$$f(x) = 9x + 7$$

$$g(x) = -10x + 8$$

$$f^{-1}(y) = \frac{y-7}{9}$$

$$g^{-1}(y) = \frac{y-8}{-10}$$

$$\text{So } (g \circ f)^{-1}(y) = \frac{\left(\frac{y-8}{-10}\right) - 7}{9}$$

4. There are two functions,  $h(x)$  and  $L(z)$  defined by tables below.

$x$	2	3	4	7	5
$h(x)$	-2	-1	3	2	4

$z$	-3	-1	2	3	8
$L(z)$	2	1	3	5	-1

Calculate the following values.

a)  $(L \circ h)(3)$

$$= L(h(3))$$

$$= L(-1)$$

$$= 1$$

b)  $(h^{-1} \circ L^{-1})(3)$

$$= h^{-1}(L^{-1}(3))$$

$$= h^{-1}(2)$$

$$= 7$$

c)  $(L^{-1} \circ h)(3)$

$$= L^{-1}(h(3))$$

$$= L^{-1}(-1)$$

$$= 8$$

d)  $(h \circ L)^{-1}(3)$

$$= (L^{-1} \circ h^{-1})(3)$$

$$= L^{-1}(h^{-1}(3))$$

$$= L^{-1}(4) = 3$$

