Classwork 15, MATH 1113 Harrison Chapman

Name: Key.

8:00 or 9:30

1. A drug is eliminated from the body through sweat. Suppose that for a dose of 10 milligrams, the amount $A(t)$ remaining in the body t hours later is given by $A(t) = 10(0.8)^t$ and that in order for the drug to be effective, at least 2 milligrams must be in the body.

a) Determine when 2 milligrams is left in the body. $(A(t) = 2)$

$$2 = 10(0.8)^t$$

$$\ln(2) = \ln(10(0.8)^t)$$

$$\ln(2) = \ln(10) + t\ln(0.8)$$

$$\boxed{\frac{\ln(2) - \ln(10)}{\ln(0.8)} = t \text{ hours}}$$

b) What is the half-life of the drug? $(A(t) = 5)$

$$5 = 10(0.8)^t$$

$$\ln(5) = \ln(10(0.8)^t)$$

$$\ln(5) = \ln(10) + t\ln(0.8)$$

$$\boxed{\frac{\ln(5) - \ln(10)}{\ln(0.8)} = t}$$

c) What is the decay constant?

$$A(t) = A_0 e^{-\lambda t}$$

$A_0 = 10$, $e^{-\lambda t} = (0.8)^t$

$$0.8^t = e^{-\lambda t}$$

$$\ln((0.8)^t) = \ln(e^{-\lambda t})$$

$$t\ln(0.8) = -\lambda t$$

$$\boxed{-\ln(0.8) = \lambda}$$

2. The energy E (in ergs) released during an earthquake of magnitude R is approximated by the formula, $\log E = 11.5 + (1.5)R$. Find the energy released from an earthquake which reads $R = 9.0$ on the Richter scale.

$$E = 10^{(11.5 + (1.5)R)}$$

$$\boxed{E = 10^{(11.5 + (1.5)(9.0))} \text{ ergs.}}$$

$\swarrow N_0$ $\qquad\qquad\qquad\qquad \swarrow (3, 600)$

3. A pond is stocked with 1000 trout. After 3 months, only 600 trout remain. Find a formula which predicts how much trout will be present after t months.

$$N(t) = N_0 e^{-kt}$$

$$N(t) = 1000 e^{-kt}$$

$$600 = 1000 e^{-k(3)}$$

$$\ln(600) = \ln(1000 e^{-k(3)})$$

$$\ln(600) = \ln(1000) - 3k$$

$$\frac{\ln(1000) - \ln(600)}{3} = k$$

$$\boxed{N(t) = 1000 e^{-\left(\frac{\ln(1000) - \ln(600)}{3}\right)t}}$$

How long will it be until there are only half as many trout as began in the pond?

$$500 = 1000 e^{-\left(\frac{\ln(1000) - \ln(600)}{3}\right)t}$$

$$\frac{1}{2} = e^{-\left(\frac{\ln(1000) - \ln(600)}{3}\right)t}$$

$$\ln\left(\tfrac{1}{2}\right) = -\left(\frac{\ln(1000) - \ln(600)}{3}\right)t$$

$$\boxed{\frac{\ln\left(\frac{1}{2}\right)}{-\left(\frac{\ln(1000) - \ln(600)}{3}\right)} = t \text{ months}}$$

4. Alice's bank offers interest at a rate of 5% compounded continuously. Bob's bank offers interest at a rate r compounded monthly. Craig invests an equal amount of money at each of Alice and Bob's banks, and after two years Craig has the same amount of money in both accounts. Find r.

$$A = Pe^{0.05t} \qquad B = P\left(1 + \frac{r}{12}\right)^{12t}$$

(when $t = 2$, $A = B$)

$$Pe^{0.05(2)} = P\left(1 + \frac{r}{12}\right)^{12(2)}$$

$$e^{0.1} = \left(1 + \frac{r}{12}\right)^{24}$$

$$\left(e^{0.1}\right)^{\frac{1}{24}} = 1 + \frac{r}{12}$$

$$\boxed{12\left(\left(e^{0.1}\right)^{\frac{1}{24}} - 1\right) = r}$$

If Craig plans on maximizing his interest over the span of 5 years, which bank should he invest his savings in, and why?

It doesn't matter. They always both pay out exactly the same!

$$A = Pe^{0.05t} \qquad B = P\left(1 + \frac{12\left(\left(e^{0.1}\right)^{\frac{1}{24}} - 1\right)}{12}\right)^{12t} = P\left(\left(e^{0.1}\right)^{\frac{1}{24}}\right)^{12t}$$

$$= Pe^{0.05t}$$