Classwork 14, MATH 1113 Harrison Chapman

Name: Key

8:00 or 9:30

1. If a certain bacteria population quadruples in 3 hours, determine the time T in hours that it takes the population to triple.

$$4P_0 = P_0 e^{k3}$$
$$4 = e^{k3}$$
$$\ln(4) = k3$$
$$\frac{\ln(4)}{3} = k$$

$$3P_0 = P_0 e^{kt}$$
$$3 = e^{kt}$$
$$\ln(3) = kt$$
$$\frac{\ln(3)}{k} = t$$

$$\boxed{T = \frac{\ln(3)}{\left(\frac{\ln(4)}{3}\right)} = t \text{ hours}}$$

2. 86% of a radioactive material remains after 20 days.

a) Find the decay constant.

$$0.86 = e^{-k20}$$
$$\ln(0.86) = -k20$$
$$\boxed{\frac{\ln(0.86)}{-20} = k}$$

b) Find the time T in days after the initial measurement when 44% of the original amount of material remains.

$$0.44 = e^{-kT}$$
$$\ln(0.44) = -kT$$
$$\frac{\ln(0.44)}{-k} = T$$

$$\boxed{T = \frac{\ln(0.44)}{\left(\frac{\ln(0.86)}{20}\right)} \text{ days}}$$

3. Alice invests \$2000 at Bob's bank and \$4000 at Charlie's bank. Bob compounds interest continuously at a nominal rate of 8%. Charlie compounds continuously at a nominal rate of 6%.
In how many years will the two investments be worth the same amount? How much will they each be worth then?

$$\underbrace{2000e^{0.08t}}_{\text{Bob}} = \underbrace{4000\,e^{0.06t}}_{\text{Charlie}}$$

$$\boxed{A = \$2000e^{0.08\left(\frac{\ln(4000)-\ln(2000)}{0.02}\right)}}$$

$$\ln\left(2000\,e^{0.08t}\right) = \ln\left(4000\,e^{0.06t}\right)$$

$$\ln(2000) + \ln\left(e^{0.08t}\right) = \ln(4000) + \ln\left(e^{0.06t}\right)$$

$$\ln(2000) + 0.08t = \ln(4000) + 0.06t$$

$$0.02t = \ln(4000) - \ln(2000)$$

$$\boxed{t = \frac{\ln(4000)-\ln(2000)}{0.02} \text{ years}}$$

4. Air pressure $p(h)$ in lb/in^2 at an altitude of h feet above sea level is approximated by the formula $p(h) = 14.7e^{-0.0000385h}$.
At approximately what altitude h is the air pressure 14 lb/in^2?

$$14 = 14.7e^{-0.0000385h}$$

$$\ln(14) = \ln(14.7) + \ln\left(e^{-0.0000385h}\right)$$

$$\ln(14) - \ln(14.7) = -0.0000385h$$

$$\boxed{\frac{\ln(14)-\ln(14.7)}{-0.0000385} = h \text{ feet}}$$

5. Alice makes an initial investment on January 1, 2000 into a bank account on that compounds continuously at an unknown rate.
On January 1, 2003, the balance was \$270.00. On January 1, 2014, the balance was \$400.00.

a) Determine the interest rate.

$$270 = Pe^{r(3)} \longrightarrow 270 = \frac{400}{e^{r(14)}} e^{r(3)}$$

$$400 = Pe^{r(14)} \rightarrow \frac{400}{e^{r(14)}} = P$$

$$\ln(270) = \ln\left(\frac{400}{e^{r(14)}} e^{r(3)}\right)$$

$$\ln(270) = \ln\left(\frac{400}{e^{r(14)}}\right) + \ln\left(e^{r(3)}\right)$$

$$\ln(270) = \ln(400) - \ln\left(e^{r(14)}\right) + r(3)$$

$$\ln(270) - \ln(400) = 3r - 14r$$

$$\ln(270) - \ln(400) = -11r$$

$$\boxed{\frac{\ln(270) - \ln(400)}{-11} = r}$$

b) Determine the initial investment.

$$P = \frac{400}{e^{r(14)}}$$

$$\boxed{P = \$\frac{400}{e^{\left(\frac{\ln(270) - \ln(400)}{-11}\right) \cdot 14}}}$$

6. A bacteria population begins with 540 bacteria present and grows exponentially. Each bacterium divides into 2 organisms every 35 minutes.

a) Find the size of the population after 4 hours.

$$2 = e^{k(35)}$$

$$\ln(2) = k(35)$$

$$\frac{\ln(2)}{35} = k$$

4 hours = 240 min

$$\boxed{P(240) = 540e^{\left(\frac{\ln(2)}{35}\right)(240)}}$$

b) After how many minutes will the population triple?

$$3 = e^{\left(\frac{\ln(2)}{35}\right)T}$$

$$\ln(3) = \left(\frac{\ln(2)}{35}\right)T$$

$$\boxed{T = \frac{\ln(3)}{\left(\frac{\ln(2)}{35}\right)} \text{ minutes}}$$