Classwork 11, MATH 1113 Harrison Chapman

Name: Key.

8:00 or 9:30

1. The effective yield (or effective annual interest rate) for an investment is the simple interest rate that would yield at the end of one year the same amount as is yielded by the compounded rate that is actually applied. Approximate, to the nearest 0.01%, the effective yield corresponding to an interest rate of 8% per year compounded quarterly and continuously.

R

(A) (B)

Simple: $n=1$ $\quad A = P\left(1+\frac{R}{1}\right)^{1t}$

$r = 0.8 \quad t=1$

(A) Quartely. $n=4$

$$A = P\left(1+\frac{r}{4}\right)^{4t} \rightsquigarrow P\left(1+\frac{0.8}{4}\right)^4 = P(1+R)$$

$$\left(1+\frac{0.8}{4}\right)^4 = 1+R$$

$$\boxed{\left(1+\frac{0.8}{4}\right)^4 - 1 = R} \quad (A)$$

(B) Continuously "Pert"

$$A = Pe^{rt}$$

$$\rightsquigarrow Pe^{0.8} = P(1+R) \to e^{0.8} = 1+R \quad \boxed{e^{0.8}-1 = R} \quad (B)$$

2. Find the domain of the function $f(x) = \sqrt{1-e^{2x+5}}$.

$$1 - e^{2x+5} \geq 0$$

$$1 \geq e^{2x+5}$$

$$e^0 \geq e^{2x+5}$$

$$0 \geq 2x+5$$

$$-5 \geq 2x$$

$$-\frac{5}{2} \geq x$$

$$\boxed{\left(-\infty, \frac{-5}{2}\right]}$$

3. Find the domain of the function $f(x) = \dfrac{1}{4^x - 16}$.

$$4^x - 16 \neq 0$$

$$4^x \neq 16$$

$$4^x \neq 4^2$$

$$x \neq 2$$

$$\boxed{(-\infty, 2) \cup (2, \infty)}$$

4. Find an exponential function of the form $f(x) = ba^{-x} + c$ with horizontal asymptote $y = 60$, y-intercept 405, and passes through the point $P(1, 465/2)$.

$c = 60$

$(0, 405)$

$405 = ba^{-0} + 60$

$345 = ba^{0}$

$345 = b$

$\frac{465}{2} = 345(a^{-1}) + 60$

$\frac{345}{2} = 345a^{-1}$

$\frac{1}{2} = a^{-1}$

$2 = a$

$\cancel{f(x) = 345(2)^{-x}}$

$\boxed{f(x) = 345(2^{-x}) + 60}$

5. The population density of sparrows in a town is approximated by the function,

$$P(t) = \frac{18000e^{5t}}{1 + 7e^{5t}},$$

where t is the time in days. Determine the long-term population density (what number the population density approaches after a very long time).

when t is very large,

$$P(t) \approx \frac{18000e^{5t}}{7e^{5t}} = \boxed{\frac{18000}{7}}$$