Math 317: Homework 9

Due Friday, April 19, 2019

1. (32.2) Let the function f be defined as,

$$f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

   a. Compute the upper and lower Darboux integrals for f on the interval $[0, b]$.

   b. Is f integrable on the interval $[0, b]$?

2. Let f be a bounded function on $[a, b]$. Suppose that f^2 is integrable. Does it follow that f is also integrable? If so, prove it. If not, provide a counterexample.

3. (32.6) Let f be a bounded function on $[a, b]$. Suppose there exist sequences (U_n) and (L_n) of upper and lower Darboux sums for f such that $\lim(U_n - L_n) = 0$. Show f is integrable on $[a, b]$ and that $\int_a^b f = \lim U_n = \lim L_n$.

4. (32.7) Let f be integrable on $[a, b]$ and suppose g is a function on $[a, b]$ so that $g(x) = f(x)$ except perhaps at finitely many x in $[a, b]$.

   Show that g is integrable on $[a, b]$ and that $\int_a^b f = \int_a^b g$. *Hint*: First reduce to the case where $f(x) = 0$ for all x.