Math 317: Homework 6

Due Friday, March 15, 2019

1. (19.4)

   a. Prove that if f is uniformly continuous on a bounded set S, then f is a bounded function on S. (*Hint*: Try proof by contradiction.)

   b. Explain why (a) gives a proof that $1/x^2$ is not uniformly continuous on $(0, 1)$.

2. (23.1, 23.2) For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

   a. $\sum \left(\frac{x}{n}\right)^n$

   b. $\sum \left(\frac{n^3}{3^n}\right) x^n$

   c. $\sum \left(\frac{3^n}{n4^n}\right) x^n$

   d. $\sum x^{n!}$

3. (24.4) For $x \in [0, \infty)$, let $f_n(x) = \dfrac{x^n}{1 + x^n}$.

   a. Find $f(x) = \lim f_n(x)$, the pointwise limit of (f_n).

   b. Determine whether $f_n \to f$ uniformly on $[0, 1]$.

   c. Determine whether $f_n \to f$ uniformly on $[0, \infty)$.