Math 317: Homework 4

Due Friday, March 1, 2019

1. (12.4) Show $\limsup(s_n + t_n) \le \limsup s_n + \limsup t_n$ for bounded sequences (s_n) and (t_n). You may use the results of Exercise 9.9 from the book without first proving them. *Hint:* First show

$$\sup\{s_n + t_n : n > N\} \le \sup\{s_n : n > N\} + \sup\{t_n : n > N\}$$

   and then apply Exercise 9.9(c).

2. (12.12) **This question is worth double points**. Let (s_n) be any sequence of non-negative real numbers. Let (σ_n) be defined by,

$$\sigma_n = \frac{s_1 + s_2 + \cdots + s_n}{n}.$$

   a. Show

$$\liminf s_n \le \liminf \sigma_n \le \limsup \sigma_n \le \limsup s_n$$

   Hint: For the last inequality, show first that $M > N$ implies

$$\sup\{\sigma_n : n > M\} \le \frac{1}{M}(s_1 + s_2 + \cdots + s_N) + \sup\{s_n : n > N\}$$

   b. Show that if $\lim_{n\to\infty} s_n$ exists, then (σ_n) converges and $\lim_{n\to\infty} \sigma_n = \lim_{n\to\infty} s_n$

   c. Give an example of a sequence (s_n) so that (σ_n) converges but (s_n) does not.

3. (14.1–4) For each of the following, determine whether the given series converges and justify your answer.

   a. $\sum \frac{n^3}{3^n}$

   b. $\sum \frac{1}{2^n+n}$

   c. $\sum \frac{n!}{n^n}$

4. (14.8) Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges. *Hint:* Show $\sqrt{a_n b_n} \le a_n + b_n$ for all n.