Math 317: Homework 3

Due Friday, February 15, 2019

1. (10.5) Complete the proof of Theorem 10.4 by showing that if (s_n) is an unbounded decreasing sequence, that $\lim s_n = -\infty$.
2. Let (s_n) be a sequence defined by $s_1 = \sqrt{2}$ and $s_{n+1} = \sqrt{2 + \sqrt{s_n}}$. Show that (s_n) converges to a real number. You may use that if $0 \leq a \leq b$ then $0 \leq \sqrt{a} \leq \sqrt{b}$. (**Hint**: Use the Monotone Convergence Theorem.)
3. (10.8) Let (s_n) be an increasing sequence of positive numbers. Let (σ_n) be defined by,

$$\sigma_n = \frac{s_1 + s_2 + \cdots + s_n}{n}.$$

   Prove that (σ_n) is in increasing sequence. (The σ_n are called *Cesaro means*)
4. For each of the following sequences, find the lim inf and the lim sup.
   a. $a_n = \frac{(-1)^n}{n}$
   b. $b_n = n \cos\left(\frac{n\pi}{4}\right)$
   c. $c_n = (-1)^n + \frac{1}{n}$
5. (11.8) Prove that $\liminf s_n = -\limsup(-s_n)$ for any sequence s_n. You may use the result of exercise 5.4 without proving it.