Math 317: Homework 10

Due Friday, April 26, 2019

1. (33.2) Let S be a nonempty bounded subset of $\mathbb{R}$. For fixed $c > 0$, let $cS = \{cs : s \in S\}$. Show that $\sup(cS) = c \cdot \sup(S)$ and $\inf(cS) = c \cdot \inf(S)$.

2. (33.6) Let f be integrable on $[a, b]$. Prove that, for any subset $S \subseteq [a, b]$ we have

$$M(|f|, S) - m(|f|, S) \leq M(f, S) - m(f, S)$$

   Hint. For $x_0, y_0 \in S$, we have $|f(x_0)| - |f(y_0)| \leq |f(x_0) - f(y_0)| \leq M(f, S) - m(f, S)$.

3. (33.5) Show that $\left|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\, dx\right| \leq \frac{16\pi^3}{3}$.

4. (33.8) Let f and g be integrable functions on $[a, b]$.

   a. It is a fact (see Exercise 33.7) that if h is integrable on $[a, b]$, then so is h^2. Prove that fg is integrable on $[a, b]$. *Hint.* Use that $4fg = (f + g)^2 - (f - g)^2$.

   b. Show that $\max(f, g)$ and $\min(f, g)$ are integrable on $[a, b]$. You may use the results of Excercise 17.8 without proof.

5. (33.10) Let f be the function,

$$f(x) = \begin{cases} \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

   Prove that f is integrable on $[-1, 1]$. *Hint.* See Excercise 33.11(c) and its solution in the textbook. (Why can we not apply the Dominated Convergence Theorem to prove that f is integrable?)