Linear Algebra I: Homework 8

Due Friday, April 13, 2018

1. Diagonalize the following matrices:

   a. The matrix,

   $$C = \begin{pmatrix} -1 & -5 \\ 4 & 7 \end{pmatrix}$$

   b. The matrix,

   $$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

2. Given the following information about the 4×4 real matrix A, answer whether: (I) A diagonalizes, (II) A does not diagonalize, or (III) there is not enough information to tell.

   a. A is invertible.

   b. A has 4 distinct real eigenvalues.

   c. The two eigenspaces of A are 3-dimensional and 1-dimensional.

   d. $A^5 = 0$.

3. Show that the following matrix is orthogonal:

   $$M = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix}$$

   Note: A prior version of this question had a typo—if you prove that that old matrix was NOT orthogonal, you are also eligible for full credit!

4. A matrix Q has characteristic polynomial $\det(\lambda I - Q) = p_Q(\lambda) = \lambda^3 - 2\lambda^2 + \lambda + 5$.

   a. Calculate $\det(Q)$.

   b. Q has exactly one real eigenvalue. Does Q diagonalize over $\mathbb{C}$?

5. A matrix R has characteristic polynomial $\det(\lambda I - R) = p_R(\lambda) = \lambda^2 - 7\lambda + 12$.

   a. Calculate $\det(R)$.

   b. Calculate $\mathrm{tr}(R)$.