Linear Algebra I: Homework 7

Due Friday, April 6, 2018

1. Find bases for the eigenspaces (and corresponding eigenvalues) of the following matrices.

   a. The matrix,

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

   b. The matrix,

$$\begin{pmatrix} 9 & -8 & 6 & 3 \\ 0 & 9 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix}$$

2. The second derivative $\frac{d^2}{dx^2}$ is a linear operator on the space of functions. Let $\omega > 0$. Show that $\sin(\sqrt{\omega}x)$ and $\cos(\sqrt{\omega}x)$ are eigenvectors of $\frac{d^2}{dx^2}$, and find their eigenvalues.

3. Find a matrix B that has eigenvalues 2, -1, 1, and for which,

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},$$

   are their corresponding eigenvectors.

4. If n is a positive integer, use diagonalizations to find A^n. It's fine (encouraged in fact!) to leave your answer factored:

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

5. Let R be a transition matrix for a Markov chain,

$$R = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & 0 & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{3} & \frac{1}{3} & \frac{2}{5} \\ \frac{1}{4} & \frac{1}{6} & \frac{2}{3} & \frac{1}{5} \\ 0 & \frac{1}{6} & 0 & \frac{1}{5} \end{pmatrix}$$

   All entries of R^2 are positive. Approximate,

$$R^{100,002,018}\begin{pmatrix}\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\end{pmatrix}.$$