Linear Algebra I: Homework 6

Due Friday, March 9, 2018

1. Answer whether the following are **subspaces** of the provided vector space.

   a. For an $m \times n$ matrix M with $m \neq n$, is the set $\ker(M)$ a subspace of $\mathbb{R}^n$?

   b. For an $m \times n$ matrix M with $m \neq n$, is the set $\mathrm{Im}(M)$ a subspace of $\mathbb{R}^n$?

   c. Is the set of rational functions $h(x) = f(x)/g(x)$ (with $g(x) \neq 0$) a subspace of the vector space of all functions?

   d. For any 3×4 matrix M is the set of solutions to the equation

   $$M\vec{x} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

   a subspace of $\mathbb{R}^4$?

2. a. How many ways are there to write the vector

   $$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

   as a linear combination of the vectors:

   $$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

   b. If $ad - bc \neq 0$, how many ways are there to write the vector $\vec{x} \in \mathbb{R}^2$ as a linear combination of the vectors:

   $$\begin{pmatrix} a \\ c \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} b \\ d \end{pmatrix}$$

3. a. Find a set S of vectors so that

   $$\mathrm{span}\,(S) = \mathrm{Im} \begin{pmatrix} 1 & 2 & 2 & 3 \\ 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

   b. Find a set S of vectors so that

$$\operatorname{span}(S) = \ker \begin{pmatrix} 1 & 2 & 2 & 3 \\ 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

4. Suppose that the vectors $\vec{u}, \vec{v}, \vec{w}$ in $\mathbb{R}^n$ are all *nonzero* and orthogonal to each other. Is the set $\{\vec{u}, \vec{v}, \vec{w}\}$ linearly dependent? **Hint:** What happens if $\vec{u} = a\vec{v} + b\vec{w}$?

5. Answer whether the following sets of vectors are linearly independent.

   a.

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \right\}$$

   b.

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\}$$