## Linear Algebra I: Homework 2

Due Friday, February 2, 2018

1. The following augmented matrices correspond to a linear system which has already been reduced by Gaussian elimination.

   For each part, describe all of the solutions to the system described by the augmented matrix. (You may express your answer in any way that you feel comfortable (as $n$-tuples, vectors, variable assignments...), but if you use variable names, make sure to let me know what they mean).

   a.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{array}\right)$$

   b.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

   c.

$$\left(\begin{array}{ccccc|c} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

   d.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right)$$

2. Put the following linear system into an augmented matrix, reduce it using Gaussian elimination, and express the solution.

$$\begin{aligned} 2a + 2b + 2c &= 0 \\ -2a + 5b + 2c &= 1 \\ 8a + b + 4c &= -1 \end{aligned}$$

3. In each of the following parts, each asterisk $*$ corresponds to any real number (they might all be different). If possible, answer whether the system represented by the

augmented matrix has **(I) no solutions**, **(II) one unique solution**, **(III) infinitely many solutions**, or that there is **(IV) not enough information** to tell.

a.

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & 0 \\ 0 & 0 & 1 & * \end{array}\right)$$

b.

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & * \end{array}\right)$$

c.

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{array}\right)$$

d.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & * & * & * \end{array}\right)$$

4. Under what conditions on the real numbers $a$ and $b$ will the following linear system have **no solutions**, **one solution**, and **infinitely many solutions**?

$$\begin{aligned} 2x - 3y &= a \\ 4x - 6y &= b \end{aligned}$$

5. a. The equation for a parabola is $ax^2 + bx + c = y$. Plug each of the three points in $xy$-coordinates $(0, 3)$, $(1, 2)$, and $(2, 4)$ into the parabola equation one at a time to obtain a linear system of three equations.

   b. How many solutions are there to your system of linear equations from part (a)? You may use what you know about parabolas to answer this question.