Math 301 Midterm Exam #1 Practice Problems

1. Your answer to the following questions should just be a number. If your number is naturally written as a product, using for instance factorials or binomial coefficients, please leave it that way!

   You should explain your reasoning as best you can: Correct answers will receive full credit, but I can only award partial credit if you show your process.

   (a) What is the coefficient of x^6y^{29} in $(x+y)^{2019}$?

   (b) What is the coefficient of $x^6y^{29}z^4$ in $(1+x+y+z)^{2019}$?

   (c) How many anagrams are there of the word AFTEREFFECTS?

   (d) How many anagrams are there of the word AFTEREFFECTS that have its vowels A and E in alphabetical order? (For example, AFTEREFFECTS counts but EFFECTFASTER will not)

   (e) How many anagrams of the word AFTEREFFECTS don't have any two E's in a row? (For example, FASTEREFFECT counts but FREESTAFFECT will not)

   (f) How many ways are there to give 20 identical pennies and 30 identical quarters to 8 distinguishable people?

   (g) You have a list of 500 cities to consider that includes Tokyo, Seattle, and Paris. How many top 10 "best city to live in" lists can you make using these cities where Seattle ranks 5th, Paris ranks 7th, and Tokyo ranks 9th?

   (h) You have a list of 500 cities to consider that includes Tokyo, Seattle, and Fort Collins. How many top 10 "best city to live in" lists can you make using these cities have Tokyo, Seattle, and Fort Collins as the top 3 cities to live in?

   (i) How many binary numbers with 10 digits–that is numbers with 10 of the digits in the set $\{0,1\}$–have no adjacent 0's? (For example 1110110110 counts but 1100110001 does not)

   (j) How many students must be in next semester of Math 301 in order for us to be sure that at least 3 students share a birth month?

   (k) How many ways are there to climb a set of 10 stairs if each step I can climb either 1, 2, or 3 stairs?

2. Give a recurrence relation for the numbers K_n, the number of words of n letters from the alphabet $\{a,b,c\}$ with no three-in-a-row runs of any one letter. (For example, *aabbabc* is allowed but *aacbbba* is not)

3. How many ways are there to tile a 4×4 chessboard with non-overlapping dominos that are exactly of the size 2×1?

4. How many ways are there to place 8 identical non-attacking rooks on an 8×8 chessboard if we remove two opposite corners?

5. Prove that $n! \geq 2^n$ for all $n \geq 4$.

6. Prove that $2^0 + 2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$.

7. Suppose that you have a set of 42 positive integers. Prove that at least 5 of these integers have the same 1's digit.