Linear Algebra I: Homework 7

Due Friday, October 13, 2017

1. Consider the set of vectors in $\mathbb{R}^3$:

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix} \right\}$$

   a. Find a vector in S which can be expressed as a linear combination of the other vectors in S.

   b. Make a new set of vectors T by removing your vector from part (a) from S. Is T linearly independent? Explain.

   c. Find a vector in T which can be expressed as a linear combination of the other vectors in T.

   d. Make a new set of vectors U by removing your vector from part (c) from T. Is U linearly independent? Explain.

2. A **unit vector** is a vector whose magnitude is 1.

   a. Describe all unit vectors $\vec{x}$ in $\mathbb{R}^2$.

   b. For which unit vectors $\vec{x}$ is

$$S = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{x} \right\}$$

   a basis for $\mathbb{R}^2$?

3. a. Find a basis for the vector space of diagonal 2×2 matrices.

   b. An **upper triangular** matrix is a matrix whose entries below diagonal entries are all 0. Find a basis for the vector space of upper triangular 2×2 matrices.

4. Consider the two bases for $\mathbb{R}^2$;

$$B = \left\{ \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right\} \qquad C = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right\}$$

   a. Find a matrix M that changes column vectors for basis B into column vectors for basis C.

   b. Find a matrix N that changes column vectors for basis C into column vectors for basis B.

5. Is 5 an eigenvalue of the matrix:

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}?$$

Explain your answer.