Linear Algebra I: Homework 4

Due Friday, September 15, 2017

1. Let $\vec{v}$ be the vector in $\mathbb{R}^4$ which points from $P(1,-1,0,3)$ to $Q(0,-2,-3,-3)$.

   a. Express $\vec{v}$ as a column vector.
   b. Find the magnitude of $\vec{v}$.
   c. Find the angle from $\vec{v}$ to the vector,

$$\vec{w} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

2. The matrix R_θ

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

   has a nice graphical explanation. If $\vec{v}$ is a 2-vector in $\mathbb{R}^2$, the vector $R_\theta\vec{v}$ (that is, the product of the matrix multiplication) has the same length as $\vec{v}$, but has been rotated by θ degrees counterclockwise (θ can be any angle).

   Let $\vec{v}$ be the 2-vector

$$\vec{v} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

   Without actually computing the vector $R_{\pi/2}\vec{v}$, compute the following:

   a. The dot product,

$$\vec{v} \cdot \left(R_{\pi/2}\vec{v}\right).$$

   b. The magnitude,

$$\|R_{\pi/2}\vec{v}\|.$$

3. Let $\vec{r_0}$ be a fixed vector in $\mathbb{R}^2$. For each part, describe *in words* the set of all vectors $\vec{r}$ that satisfy the stated condition. **Hint**: Think about nice shapes. An answer which just re-writes the math in English will not receive full credit.

   a. $\|\vec{r} - \vec{r_0}\| = 1$.
   b. $\|\vec{r} - \vec{r_0}\| \geq 1$.

4. Explain why the line of 3-vectors,

$$L = \left\{ \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \;\middle|\; t \in \mathbb{R} \right\}$$

is not a vector space.

5. Does there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that,

$$T\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

$$T\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

and,

$$T\left(\begin{pmatrix} 5 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 12 \\ 1 \end{pmatrix}?$$

Justify your answer.