

Linear Algebra I: Homework 12

Due Monday, December 4, 2017

Notice: This final homework assignment is due after our final midterm, but the questions on it cover material that will be on the test. I encourage you to work these problems before the test, although you don't need to have a set of solutions written up until after.

1. Let P_n be the vector space of polynomials in x of degree at most n , and let $\frac{d}{dx}$ be the derivative (a linear transformation on P_n).
 - a. What is the rank of $\frac{d}{dx}$?
 - b. What is the nullity of $\frac{d}{dx}$?
2. Let P_4 be the vector space of polynomials in x of degree at most 4. Then $\langle f, g \rangle = \int_0^{10} 4f(x)g(x) dx$ is an inner product on P_4 . If B is an orthonormal basis for P_4 under this inner product, compute:

$$\left\langle \begin{pmatrix} 1 \\ -1 \\ 3 \\ 2 \end{pmatrix}_B, \begin{pmatrix} 6 \\ 0 \\ -1 \\ 4 \end{pmatrix}_B \right\rangle$$

3. Find an orthonormal basis for the kernel of the matrix M :

$$M = \begin{pmatrix} 2 & 2 & -2 & 3 \\ 1 & -6 & -1 & -9 \end{pmatrix}$$

4. For the following parts, a matrix or linear transformation is described. Explain whether or not it is **invertible**.
 - a. The matrix K diagonalizes as

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

- b. The linear transformation $L : \mathbb{R}^n \rightarrow \mathbb{R}^n$ scales volumes of hypercubes by a factor of $1/2$.
 - c. The rank of the linear transformation $H : \mathbb{R}^7 \rightarrow \mathbb{R}^7$ is 5.
 - d. The linear transformation $Q : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is surjective.
5. Find the component of the velocity vector,

$$\vec{v} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

in the direction as the vector,

$$\vec{w} = \begin{pmatrix} 10 \\ 3 \\ 2 \end{pmatrix}$$

6. Find the least squares solution to the equation,

$$\begin{pmatrix} 1 & -1 \\ 3 & 0 \\ 2 & 1 \end{pmatrix} \vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$