Linear Algebra I: Homework 11

Due Friday, November 17, 2017

1. a. Find the explicit change of basis matrix from the standard basis E of $\mathbb{R}^3$ to the orthonormal basis B

$$\left(\begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} \end{pmatrix} , \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} \end{pmatrix} , \begin{pmatrix} 0 \\ \frac{-1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} \end{pmatrix} \right)$$

   b. Find the explicit change of basis matrix from B to E.

2. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the set of vectors,

$$\left\{ \begin{pmatrix} -3 \\ -2 \\ 4 \\ 0 \end{pmatrix} , \begin{pmatrix} 8.5 \\ 3 \\ -3 \\ 1 \end{pmatrix} , \begin{pmatrix} -1.5 \\ -12 \\ -3.5 \\ 3.5 \end{pmatrix} \right\}$$

3. Let

$$\vec{v} = \begin{pmatrix} 9 \\ -1 \\ -3 \\ 9 \end{pmatrix} .$$

   Find a basis of the subspace of $\mathbb{R}^4$ of all vectors perpendicular to $\vec{v}$

4. If U is a subspace of a vector space W, prove that $U^{\perp}$ is a subspace of W.

5. a. Find a basis for the kernel of the matrix A,

$$A = \begin{pmatrix} 6 & -3 & 6 & 3 \\ -4 & 2 & -4 & -2 \end{pmatrix} .$$

   b. Find a basis for the column space of the matrix B,

$$B = \begin{pmatrix} 3 & 5 & 1 \\ 5 & -4 & 14 \\ 5 & -4 & 14 \\ -2 & -4 & 0 \\ 4 & 9 & -1 \end{pmatrix} .$$